CONTROL OF REDUNDANT ROBOTS AT SINGULARITIES IN DEGENERATE DIRECTIONS

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Abstract

An algorithm for controlling redundant robots at singularities, when the desired motion is in a degenerate direction, is proposed. This case is not supported by other solutions, using only Jacobian information, proposed in the literature. Our algorithm uses Jacobian and Hessian matrices and it allows to choose the configuration of the robot when it exits the singularity. The algorithm was tested on a 7 degree of freedom robot.

Introduction

Let $\mathbf{q}$ be the current joint coordinates vector, of size [n,1], of a robot, and $\mathbf{x}$ a [m,1] vector of end-effector Cartesian coordinates, then the forward kinematics is:

$$\mathbf{x} = \mathbf{f}(\mathbf{q}) \tag{1}$$

The Cartesian velocity is obtained by deriving equation (1) with respect to time:

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \tag{2}$$

where $\mathbf{J}$ is the [m,n] Jacobian matrix. The inverse kinematics problem is solved by finding the joint vector $\tilde{\mathbf{q}} = \mathbf{q} + \delta\mathbf{q}$ so that $\delta\mathbf{x} = \mathbf{f}(\mathbf{q} + \delta\mathbf{q}) - \mathbf{f}(\mathbf{q})$. The state of the art relies on the assumption that the displacements $\delta\mathbf{x}$ and $\delta\mathbf{q}$ are small enough so that the following relationship holds from equation (2):

$$\delta\mathbf{x} = \mathbf{J}(\mathbf{q})\delta\mathbf{q} \tag{3}$$

If $\mathbf{J}$ is a $m \times n$ matrix, equation (3) generates a linear system of m equations and n unknown. If $n > m$, the robot is redundant and we have an infinity of solutions. However, if we are at a singular joint position, equation (3) can be solved only if $rank([\mathbf{J} \quad \delta\mathbf{x}]) = rank([\mathbf{J}])$. When this equation is not true, an approximate solution is found using the Jacobian pseudo-inverse[1] or the damped least-squares Jacobian inverse method[2,3]. At a singularity, there may be a set of directions $\delta\mathbf{x}$ that are "degenerate". A direction $\delta\mathbf{x}$ is said to be degenerate if $\mathbf{J}^{T}\delta\mathbf{x} = \mathbf{0}$. Suppose the robot is at a singularity and the desired motion is in a degenerate direction. Solutions proposed in the literature using only Jacobian information are not capable of handling this particular case. Nielsen[4] and Egeland[5] have shown that the Cartesian acceleration term provides the information required to move along the degenerate direction. The Cartesian acceleration is obtained by deriving equation (2) with respect to time:

$$\ddot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} \tag{4}$$

where:

$$\dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}}) = \left[\mathbf{H}_1(\mathbf{q})\dot{\mathbf{q}} \quad \mathbf{H}_2(\mathbf{q})\dot{\mathbf{q}} \quad \cdots \quad \mathbf{H}_m(\mathbf{q})\dot{\mathbf{q}}\right]^T \tag{5}$$

The m Hessian matrices $\mathbf{H}_k$ of size [n,n] are symmetric. We proposed in another paper[6] an algorithm to compute in real-time Hessian matrices for solving the inverse kinematics problem and so how accomplish motion in degenerate direction at singularities without trajectory generation error. This paper is organized as follows. In the first section we illustrate the algorithm for inverse kinematics at singularities in degenerate directions. In the second section we discuss the experimental results.

Inverse Kinematics

The algorithm proposed in this paper minimizes the following set of functions:

$$\min_{\delta\mathbf{q}} \quad \Phi = \|\delta\mathbf{q}\|^2 \tag{6}$$

$$\min_{\delta\mathbf{q}} \quad \Psi = \left\|\delta\mathbf{x} - \left(\mathbf{f}(\mathbf{q} + \delta\mathbf{q}) - \mathbf{f}(\mathbf{q})\right)\right\|^2 \tag{7}$$

Our idea is to take a second order Taylor expansion of the norm of the Cartesian error:

$$\hat{\Psi} = \delta\mathbf{x}^T\delta\mathbf{x} - 2\delta\mathbf{x}^T\mathbf{J}(\mathbf{q})\delta\mathbf{q} + \frac{1}{2}\delta\mathbf{q}^T\left[2\mathbf{J}^T(\mathbf{q})\mathbf{J}(\mathbf{q}) - \sum_{i=1}^{m}\delta x_i\mathbf{H}_i(\mathbf{q})\right]\delta\mathbf{q} \tag{8}$$

We define C, **B** and **E** in the following way:

$$\mathrm{C} = \delta\mathbf{x}^{\mathrm{T}}\delta\mathbf{x}, \quad \mathbf{B} = -2\delta\mathbf{x}^T\mathbf{J}(\mathbf{q}), \quad \mathbf{E} = 2\mathbf{J}^T(\mathbf{q})\mathbf{J}(\mathbf{q}) - \sum_{i=1}^{m}\delta x_i\mathbf{H}_i(\mathbf{q}) \tag{9}$$

Equation (8) gives a better approximation of the Cartesian error thanks to the term using the Hessian matrices. We test the vector **B** of equation (9). If $\mathbf{B} \neq \mathbf{0}$, we can use any of the existing algorithms. If $\mathbf{B} = \mathbf{0}$ and $\delta\mathbf{x} \neq \mathbf{0}$, $\delta\mathbf{x}$ is a degenerate vector, we cannot use those algorithms, and we have to solve the set of equations (6) and (7) written hereafter. The minimum value of Ψ is 0, thus our problem is equivalent to minimize equation (6) with the following constraint:

$$\hat{\Psi} = \mathrm{C} + \frac{1}{2}\delta\mathbf{q}^{\mathrm{T}}\mathbf{E}\delta\mathbf{q} = 0 \tag{10}$$

The Lagrange operator of equations (6) and (10) is:

$$L = \delta\mathbf{q}^T\delta\mathbf{q} + \lambda\left(C + \frac{1}{2}\delta\mathbf{q}^T\mathbf{E}\delta\mathbf{q}\right) \tag{11}$$

The necessary conditions for solving this set of equations are:

$$\frac{\partial \mathrm{L}}{\partial \delta\mathbf{q}} = \delta\mathbf{q} + \lambda\mathbf{E}\delta\mathbf{q} = 0 \tag{12}$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = \mathrm{C} + \frac{1}{2}\delta\mathbf{q}^{\mathrm{T}}\mathbf{E}\delta\mathbf{q} = 0 \tag{13}$$

If $\lambda = 0$, equation (12) implies $\delta\mathbf{q} = \mathbf{0}$ and equation (13) doesn't hold when $\delta\mathbf{x} \neq \mathbf{0}$. So λ is not null and μ is defined as $\mu = -1/\lambda$. Equation (12) can be expressed as follow: $\mathbf{E}\delta\mathbf{q} = \mu\delta\mathbf{q}$. Therefore $\delta\mathbf{q}$ is an eigen vector of **E** and μ is its eigen value. We choose a normalized vector **v** and a scalar α so that $\delta\mathbf{q} = \alpha\mathbf{v}$. By plugging this equation into equation (13), we get:

$$\mathrm{C} + \frac{1}{2}\alpha^2\mu = 0 \tag{14}$$

C is positive, therefore we have to choose a negative eigen value of **E** to verify equation (14). The solution to equation (14) is:

$$\alpha = \pm\sqrt{-2C/\mu} \tag{15}$$

The bigger is the norm of the eigen value μ, the smaller is α. Since **v** is normalized the norm of α is the norm of $\delta\mathbf{q}$. We want the smallest vector $\delta\mathbf{q}$ to satisfy equation (6), so we have to choose the smallest eigen value of **E**. The sign of α determines the configuration of the robot when it exits the singularity.

Experimental results

To simplify the explanation of the algorithm in this paper, we blocked 4 axes of the seven axis robot Mitsubishi PA-10 used in our laboratory, and obtained a three joint planar robot. In our calculations, we only consider the Cartesian position of the end-effector, therefore the robot for our example is "redundant" (see Figure 1). The links' lengths are those of the PA-10: $l_1 = 0.45$ m; $l_2 = 0.5$ m; $l_3 = 0.08$ m.

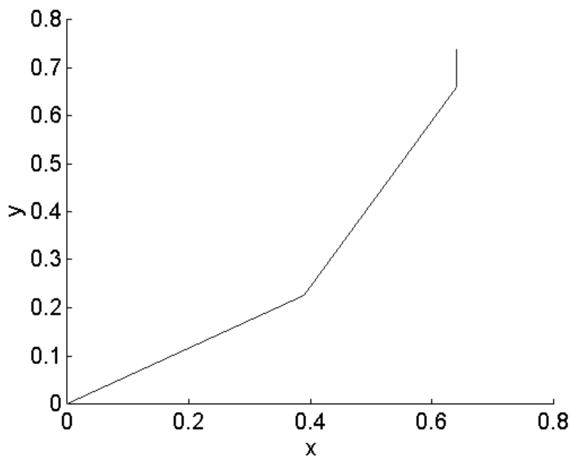

Figure 1: Three joint planar robot

If the robot is at the joint position $\mathbf{q} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ the Jacobian is of rank 1:

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 1.03 & 0.58 & 0.08 \end{bmatrix}$$

We want to compute the joint motion needed to move the robot along the x axis, for example $\delta\mathbf{x} = \begin{bmatrix}-0.001 & 0\end{bmatrix}^{\mathrm{T}}$ meters. With the methods using only Jacobian information, the result is a null joint displacement, because $\delta\mathbf{x}$ is a degenerate vector ($\mathbf{J}^{\mathrm{T}}\delta\mathbf{x} = \mathbf{0}$). In this case, we use the Hessian matrices. **E**, **B** and C are computed from equation (9). Then the eigen vectors and eigen values of **E** are found: $\mu_1 = -1.9665\,10^{-4}$; $\mu_2 = -6.7167\,10^{-5}$; $\mu_3 = 2.8060$ and $\mathbf{v}_1 = \begin{bmatrix}0.4932\ -0.8509\ -0.1808\end{bmatrix}^{\mathrm{T}}, \mathbf{v}_2 = \begin{bmatrix}0.0310\ -0.1905\ 0.9812\end{bmatrix}^{\mathrm{T}}, \mathbf{v}_3 = \begin{bmatrix}0.8694\ 0.4895\ 0.0675\end{bmatrix}^{\mathrm{T}}$
As explained earlier, we choose the eigen vector whose eigen value is the smallest (it also has to be negative). In this case, we find $\mu_1 = -1.9665\,10^{-4}$.

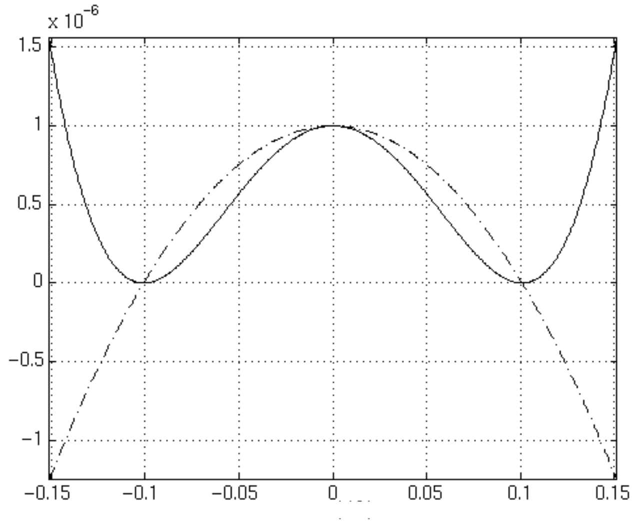

Figure 2: Cartesian error norm and its second order approximation

Figure 2 plots the functions $\Psi = \Psi(\alpha\mathbf{v})$ of equation (7) (continuous line) and its approximation $\hat{\Psi} = \hat{\Psi}(\alpha\mathbf{v})$ of equation (8) (dashed line) versus α. At $\|\delta\mathbf{q}\| = 0$, we are at the initial position, and the first derivative is null, as expected. In this example, the Hessian in the y direction is null, therefore, $\hat{\Psi} = \hat{\Psi}(\alpha\mathbf{v})$ and $\Psi = \Psi(\alpha\mathbf{v})$ are null for the same α: $\alpha = \pm\sqrt{-2C/\mu_1} = \pm 0.1008$. The absolute value of α is the norm of $\delta\mathbf{q}$ in radiants, it is equivalent to 5.77 degrees.

Generally, at a given singularity, Ψ and $\hat{\Psi}$ will not intersect at the value 0, like in Figure 2. However, it is still possible to command a motion along the vector associated with a negative eigen value of **E**, so that the norm $\hat{\Psi}$ decreases (i.e. Ψ decreases). If there are not strictly negative eigen values, the Taylor expansion needs to be taken to a higher order, and a new algorithm is needed. For the seven degree of freedom Mitsubishi robot arm, our results confirmed that there is at least one negative eigen value at each singularity.

Figure 3 shows the two configurations of the robot for $\delta\mathbf{q} = \pm[2.8496 \quad -4.9169 \quad -1.0455]^T$ degrees.

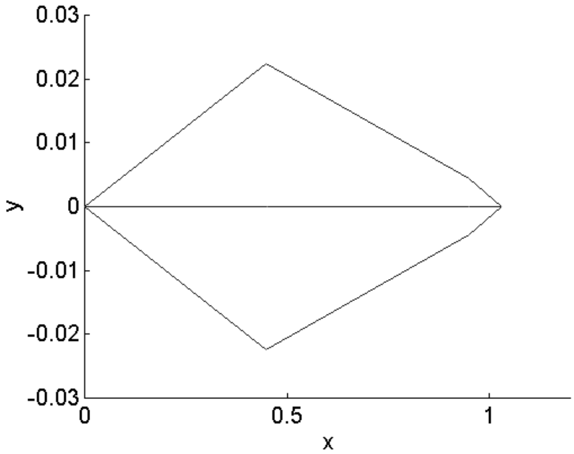

Figure 3: Movement in degenerate direction

If we choose the eigen vector corresponding to the second smallest eigen value (which is also negative, therefore it is suitable), we would find a $\delta\mathbf{q}$ with a bigger norm. We found another pair of symmetric solutions to the inverse kinematics problem.

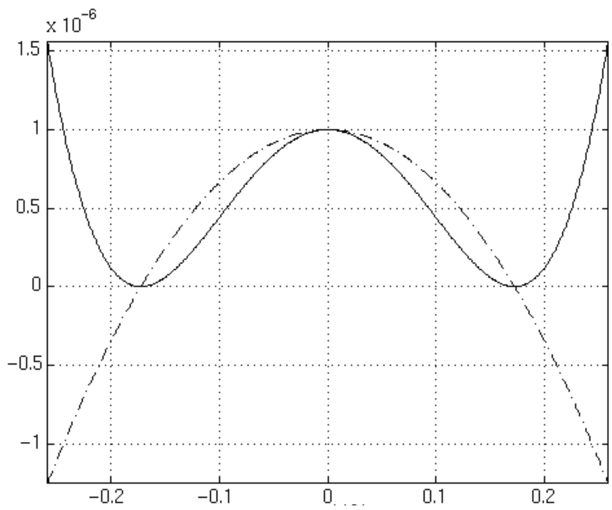

Figure 4: Cartesian error norm and its second order approximation

Figure 4 plots $\Psi = \Psi(\alpha\mathbf{v})$ of equation (7) (continuous line) and its second order approximation $\hat{\Psi} = \hat{\Psi}(\alpha\mathbf{v})$ of equation (8) (dashed line) versus α. In this case too, $\hat{\Psi} = \hat{\Psi}(\alpha\mathbf{v})$ and $\Psi = \Psi(\alpha\mathbf{v})$ are null for the same α: $\alpha = \pm\sqrt{-2C/\mu_2} = \pm 0.1726$. The absolute values of α is the norm of $\delta\mathbf{q}$ in radiants, equal to 9.89 degrees. Figure 5 shows the two joints positions of the robot for $\delta\mathbf{q} = \pm\left[0.3068 \quad -1.8930 \quad -9.7011\right]^{T}$ degrees.

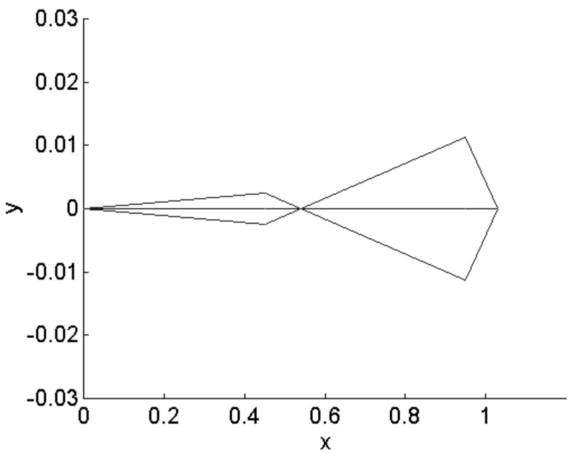

Figure 5: Movement in degenerate direction

Generally, it is better to perform the Cartesian motion with the smallest possible joint displacement. But if there is an obstacle and we cannot move the robot as in Figure 3, we can choose the second eigen value and move the robot like in Figure 5. It is therefore possible to move away from a singular configuration in a degenerate direction with little tracking error, but as the robot gets closer to the singularity, the commanded velocity should get smaller. Since $\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$, the robot will always pass the singularity at a null speed in the degenerate directions.

The algorithm proposed in the paper was tested on the PA-10 from Mitsubishi, a 7 degree of freedom robot. The Singular Value Decomposition algorithm was used to invert the Jacobian. To know if the wanted motion is along a degenerate direction, we test if $\left\|\mathbf{J}^{T}\delta\mathbf{x}\right\| \le \varepsilon_1$ and $\left\|\delta\mathbf{x}\right\| \ge \varepsilon_2$ (ε_1 and ε_2 are thresholds). There are two limitations on the norm of $\delta\mathbf{q}$:

- a Taylor expansion is used, so it is valid only for small $\delta\mathbf{q}$s. Since we take the expansion to the second degree, the range of $\delta\mathbf{q}$ is bigger than the other methods which take at best an incomplete Taylor expansion to the second order.

- the result of the computation is used on a real robot. The algorithm is therefore iterative in the sense that a new $\delta\mathbf{q}$ is calculated at every sampling time. During that time, the robot can only move a limited amount of degrees. This is imposed by the maximum acceleration and speed of the robot.

Because of these two constraints, the norm of $\delta\mathbf{q}$ is limited by a maximum value. This value was chosen during simulation runs, because there is not enough time to compute it in real time. In the case of the PA-10, we chose the limit on $\delta\mathbf{q}$ based upon the maximum joint speed of the robot. The algorithm taken at the second order was implemented on the 7 degree of freedom robot PA-10. The computation period is 20 ms on a MVME 167 processor card.

Conclusions

The algorithm described in this paper solves the problem of moving a robot in a degenerate direction from a singularity of its workspace. Other methods which solve motion through singular points using only Jacobian information are not capable of handling this particular case. The basic idea is based on taking a Taylor expansion of the norm of the function to minimize. To illustrate the algorithm, a second order expansion was performed in this paper. There may be robots for which higher orders are needed. It is, however, unlikely that this should occur: taking a second order expansion means that there is at least one pair of joints (among all the possible pairs) that can move the robot in the degenerate direction if these two joints are actuated together. By comparison, the Jacobian only calculates the differential movement due to each joint, one at a time, and then performs a linear combination of the movement.

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